ENTRI

APJ Abdul Kalam Technological University

First Semester B.Tech Degree Regular and Supplementary Examination December 2020 (2019 Scheme)

- **Course Code: MAT101**
- **Course Name: Linear Algebra and Calculus**
- **Max. Marks: 100**
- **Duration: 3 Hours**

PART A

Answer all questions, each carries 3 marks.

1. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & -1 & 3 \\ 2 & 2 & 4 & 1 \\ 5 & 6 & 7 & 5 \end{bmatrix}$.
2. What type of conic section does the following quadratic form represent?
   $Q = 17x_1^2 - 30x_1x_2 + 17x_2^2 = 128.$.
3. If $U = \frac{x^3+y^3}{x-y}$, find $\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y}$.
4. If $z = x^2y, x = t^2, y = t^3$, find $\frac{dz}{dt}$ using the chain rule.
5. Evaluate $\int_0^1 \int_0^1 \int_0^1 xyz \, dx \, dy \, dz$.

6. Use double integrals to find the volume of the solid enclosed below the plane $z = 4 - x - y$ and above the rectangle $R = \{(x, y); [cite_start]0 \leq x \leq 1, 0 \leq y \leq 2\}$.
7. Does the series $\sum_{k=1}^{\infty} (\frac{4}{5})^k$ converge? If so, find the sum.
8. Test the convergence of the series $\sum_{k=1}^{\infty} \frac{k^2}{2k^2-1}$.
9. Find the binomial series for $f(x) = \frac{1}{\sqrt{1+x}}$ up to the third-degree term.
10. Find the Maclaurin's series for $f(x) = x \cos x$ up to the third-degree term.

PART B

Answer one full question from each module, each question carries 14 marks.

Module I

11.

a) Using Gauss elimination method, find the solution of the system: x+y-z=9, 8y+6z=-6, -2x+4y-6z=40. (7 marks)

b) Find the matrix of transformation that diagonalizes the matrix $\begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$. Also, find the diagonal matrix. (7 marks)

OR

12. a) Find the values of λ and μ for which the system 2x+3y+5z=9, 7x+3y-2z=8, $2x+3y+ λz=μ has

(a) no solution

(b) unique solution

(c) more than one solution. (7 marks)

b) Find the eigenvalues and eigenvectors for the matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. (7 marks)

Module II

13. a) If $w = \sqrt{x^2 + y^2 + z^2}, x = \cos\theta, y = \sin\theta, z = \tan\theta$, find $\frac{dw}{d\theta}$. (7 marks)

b) Find the local linear approximation *L* of *f(x,y,z)=xyz* at the point *P*(1,2,3). Compute the error in approximating *f* by *L* at the point *Q*(1.001, 2.002, 3.003). (7 marks)

OR

14. a) Locate all relative extrema of $f(x,y) = x^3y^2(12 - x - y)$. (7 marks)

b) Let *f* be a differentiable function of three variables and *w=f(x-y, y-z, z-x)*. Show that $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = 0$. (7 marks)

Module III

15. a) Find the area bounded by the x-axis, *y=2x*, and *x+y=1*. (7 marks)

b) Change the order of integration and hence evaluate $\int_0^1 \int_{x^2}^{2-x} dy\, dx$. (7 marks)

OR

16. a) Find the volume bounded by the cylinder $x^2 + y^2 = 9$ and the planes *y+z=3* and *z=0*. (7 marks)

b) Find the mass and centre of gravity of the lamina in the first quadrant bounded by the circle $x^2 + y^2 = 1$ and the coordinate planes with density function *xy*. (7 marks)

Module IV

17. a) Test the convergence:

(i) $\sum_{k=1}^{\infty} \frac{k(k-1)}{(k+1)(k+2)(k+3)}$

(ii) $\sum_{k=1}^{\infty} \left(\frac{k+2}{2k-1}\right)^k$. (7 marks)

b) Test whether the following series is absolutely convergent or conditionally convergent: $\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k(k+1)}}$. (7 marks)

OR

18. a) Test the convergence of the series $1 + \frac{1.3}{1.2} + \frac{1.3.5}{1.2.3} + \frac{1.3.5.7}{1.2.3.4} + \ldots$. (7 marks)

b) Test the convergence:

(i) $\sum_{k=1}^{\infty} \frac{2}{3^k+5}$

(ii) $\sum_{k=1}^{\infty} (-1)^{k+1} \left(\frac{k}{2k+3}\right)$. (7 marks)

Module V

19. a) Find the Fourier series of

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi < x < 0 \\ 1 - \frac{2x}{\pi}, & 0 < x < \pi \end{cases}$$

. (7 marks)

b) Obtain the Fourier series of e^x in the interval (-1, 1). (7 marks)

OR

20. a) Find the Fourier series $f(x) = x^2 - 2$ in the interval (-2, 2). (7 marks)

b) Find the Fourier cosine series of $f(x) = x^2 \text{ in } 0 < x < \pi$. (7 marks)