ENTRI

APJ Abdul Kalam Technological University

First Semester B.Tech Degree Examination December 2021

(2019 Scheme)

- **Course Code: MAT101**
- **Course Name: Linear Algebra and Calculus**
- **Max. Marks: 100**
- **Duration: 3 Hours**

PART A

Answer all questions, each carries 3 marks.

1. Find the rank of the matrix $\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & -4 \\ 0 & 4 & 0 \end{bmatrix}$.

2. Find the eigenvalues of the matrix $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$. What are the eigenvalues of A^2 and A^{-1} without using its characteristic equation?

3. If $z = \frac{xy}{x^2+y^2}$, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

4. Show that the equation $u(x,t) = \sin(x - ct)$ satisfies the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$.

5. Evaluate $\int_0^3 \int_0^2 \int_0^1 xyz\, dx\, dy\, dz$.

6. Find the mass of the lamina with density $\delta(x, y) = x + 2y$ bounded by the x-axis, the line *x*=1 and the curve $y^2 = x$.

7. Find the rational number represented by the repeating decimal 5.373737…

8. Examine the convergence of $\sum_{k=1}^{\infty} \frac{k^2}{2k^2+3}$.

9. Find the Taylor series expansion of $f(x) = \sin x \text{ about } x = \frac{\pi}{2}$.

10. If *f(x)* is a periodic function with period 2π defined in $[-\pi, \pi]$, write the Euler's formulas a_0, a_n, b_n for $f(x)$.

PART B

Answer one full question from each module, each question carries 14 marks.

MODULE 1

11. a) Solve the following linear system of equations using Gauss elimination method: x+2y-z=3, 3x-y+2z=1, 2x-2y+3z=2. (7 marks)

    b) Find the eigenvalues and eigenvectors of $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$. (7 marks)

    OR

12. a) Solve the following linear system of equations using Gauss elimination method: 2x-2y+4z=0, -3x+3y-6z+5w=15, x-y+2z=0. (7 marks)

    b) Find the matrix of transformation that diagonalizes the matrix $A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$. Also write the diagonal matrix. (7 marks)

ENTRI

MODULE 2

13. a) The length and width of a rectangle are measured with errors of at most 3% and 4% respectively. Use differentials to approximate the maximum percentage error in the calculated area. (7 marks)

    b) Find the local linear approximation *L* of *f(x,y,z)=xyz* at the point *P*(1,2,3). Compute the error in approximating *f* by *L* at the point *Q*(1.001, 2.002, 3.003). (7 marks)

    OR

14. a) If *w=f(P,Q,R)* where *P=x-y*, *Q=y-z*, *R=z-x*, prove that $\frac{\partial w}{\partial x}+\frac{\partial w}{\partial y}+\frac{\partial w}{\partial z}=0$. (7 marks) b) Locate all relative extrema and saddle points of $f(x,y)=4xy-x^{4}-y^{4}$. (7 marks)

MODULE 3

15. a) Find the area bounded by the parabolas $y^2=4x \text{ and } x^2=\frac{y}{2}$. (7 marks) b) Evaluate $\int_{-2}^{2}\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} e^{-(x^2+y^2)}\,dx\,dy$ using polar coordinates. (7 marks)

    OR

16. a) Evaluate by reversing the order of integration: $\int_0^1\int_y^1 \frac{x}{x^2+y^2}\,dx\,dy$. (7 marks)

b) Use triple integral to find the volume of the solid within the cylinder $x^2 + y^2 = 9$ and between the planes *z*=1 and *x*+*z*=5. (7 marks)

MODULE 4

17. a) Test the convergence of

(i) $\sum_{k=1}^{\infty} \frac{3k^3-2k^2+4}{k^7-k^3+2}$

(ii) $\sum_{k=1}^{\infty} \frac{k^k}{k!}$. (7 marks)

b) Test whether the series $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{\sqrt{k}+1}$ is absolutely convergent or conditionally convergent. (7 marks)

OR

18. a) Test the convergence of the series

$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5} \cdots$ (7 marks)

b) Test the convergence of

` (i) $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5} \cdots$

(ii) $\sum_{k=1}^{\infty} \frac{7^k}{k!}$. (7 marks)

MODULE 5

19. a) Find the Fourier series expansion of

$f(x) = x - x^2$ in the range (-1, 1). (7 marks)

b) Obtain the half range Fourier cosine series of $f(x) = e^{-x} \text{ in } 0 < x < 2$. (7 marks)

OR

20. a) Find the Fourier series expansion of $f(x) = x^2$ in the interval $-\pi < x < \pi$. Hence show that $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots = \frac{\pi^2}{12}$. (7 marks)

b) Obtain the half range Fourier sine series of $f(x)$= $\begin{cases} x, & 0 < x < \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} < x < \pi \end{cases}$. (7 marks)