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# APJ Abdul Kalam Technological University

## Second Semester B.Tech Degree Examination July 2021

## (2019 Scheme)

- **Course Code:** MAT102
- **Course Name:** Vector Calculus, Differential Equations and Transforms
- Max. Marks: 100
- **Duration:** 3 Hours

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**PART A**

**Answer all questions, each carries 3 marks.**

1. Find the velocity and acceleration of a particle whose position vector is given by $r(t) = e^t i + e^{-t} j$ at $t = 0$.

2. If *C* is the unit circle $x^2 + y^2 = 1$ oriented counter-clockwise, find $\int_C x dx + y dy$.

3. Determine the sources and sinks of the vector field $f(x, y, z) = 2(x^3 - 2x)i - 2(y^3 - 2y)j + 2(z^3 - 2z)k$.

4. Evaluate $\iint_\sigma xzdS$ where $\sigma$ is the part of the plane $x+y+z=1$ that lies in the first octant.

5. Solve the initial value problem
$y'' + y' - 2y = 0, y(0) = 4, y'(0) = -5$.

6. Solve $y'''' + 10y'' + 9y = 0$.

7. Find the Laplace transform of $e^{-3t}\cos 2t$.

8. Find the inverse Laplace transform of $\frac{1}{(s+a)(s+b)}$.

9. Find the Fourier cosine transform of the function $f(x)$ =
$$\begin{cases} x & \text{if } 0 < x < a \\ 0 & \text{if } x > a \end{cases}$$
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ENTRI

10. Using Fourier sine integral, show that $\int_0^\infty \frac{1-\cos \pi w}{w} \sin xw dw = \begin{cases} \frac{\pi}{2} & \text{if } 0 < x < \pi \\ 0 & \text{if } x > \pi \end{cases}$.

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# PART B

**Answer one full question from each module, each question carries 14 marks.**

## Module-I

11. a) Find the directional derivative of $w(x, y, z) = x^3z - yx^2 - z^2$ at (1,1,1) in the direction of $a = 2i - j + 2k$. Also find the maximum directional derivative. (7)

    b) Show that the vector field $f(x, y) = xy^2i + x^2yj$ is conservative and find $\phi$ such that $f = \nabla\phi$. Hence evaluate $\int_{(1,2)}^{(2,4)} xy^2dx + x^2ydy$. (7)

12. a) Find the parametric equation of the tangent line to the graph $r(t)$ = $t^2i - \frac{1}{t+3}j + (4 - t^2)k$ at (4, -1, 0). (7)

b) Using line integral evaluate $\int_C x^2ydx + xdy$ where *C* is the triangular path connecting (0,0), (1,0) and (1,2) in the positive direction. (7)

**Module-II**

13. a) Use Green's theorem to evaluate $\int_C \log(1+y)dx - \frac{xy}{1+y}dy$ where *C* is the triangle with vertices (0,0), (2,0) and (0,4). (7)

b) Use Divergence theorem to find the outward flux of the vector field $F = (2x + y^2)i +$ $xyj + (xy - 2z)k$ across the surface of the tetrahedron bounded by *x*+*y*+*z*=2 and the coordinate planes. (7)

14. a) Find the flux of the vector field $F(x, y, z) = zk$ across the portion of the paraboloid $z = x^2 + y^2$ below the plane *z* = 2*y* oriented by downward unit normal. (7)

b) Use Stokes theorem evaluate $\oint_C f \cdot dr$ where $f(x, y, z) = (z - y)i + (z + x)j - (x + y)k$ and *C* is the boundary of the paraboloid $z = 9 - x^2 - y^2$ above the XY-plane with upward orientation. (7)

## Module-III

15. a) Solve using the method of undetermined coefficients: $y'' - 4y' - 5y = 4\cos 2x$. (7)

    b) Solve using the method of variation of parameters: $y'' + y = \csc x$. (7)

16. a) Solve using the method of undetermined coefficients: $y'' - 7y' + 12y = e^{2x}, y(0) = 1, y'(0) = 2$. (7)

    b) Solve the boundary value problem: $x^2y'' - 3xy' + 4y = 0, y(1) = 0, y'(1) = 1$. (7)

## Module-IV

17. a) Using convolution theorem, find the Laplace inverse of $\frac{2}{(s^2+1)(s^2+25)}$. (7)

    b) Using Laplace Transform, solve $y'' + 2y' + y = e^{-t}, y(0) = 0, y'(0) = 1$. (7)

18. a) Express in terms of unit step function and hence find the Laplace Transform of

$$f(t) = \begin{cases} t^2 & \text{if } 0 < t < 2 \\ t - 1 & \text{if } 2 < t < 3 \\ 7 & \text{if } t > 3 \end{cases}$$

. (7)

b) Find the inverse Laplace Transform of $\frac{s^2+2}{s(s^2+9)}$. (7)

**Module-V**

19. a) Find the Fourier Transform of

$$f(x) = \begin{cases} 1 & \text{if } -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

. (7)

b) Find the Fourier integral of

$$f(x) = \begin{cases} \pi - x & \text{if } 0 < x < \pi \\ 0 & \text{otherwise} \end{cases}$$

. (7)

20. a) Find the Fourier Sine Transform of

$$f(x) = \begin{cases} x & \text{if } 0 < x < 1 \\ 3 - x & \text{if } 1 < x < 3 \\ 0 & \text{if } x > 3 \end{cases}$$

. (7)

b) Represent $f(x) = e^{-kx}, x > 0, k > 0$ as a Fourier Cosine integral. (7)