Reg No.:_____ Name:_____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech Degree S8 (R,S) Exam April 2025 (2019 Scheme)

Course Code: ECT428

Course Name: OPTIMIZATION TECHNIQUES

Max. Marks: 100 **Duration: 3 Hours**

PART A

Answer all questions, each carries 3 marks. *Marks*

1 Explain the necessary and sufficient conditions for identifying local maxima and minima of a function with a single input variable and no constraints. (3)

2 A roller coaster track is modelled by the function $f(x) = x^3 - 3x^2 + 2$, $f(x)$ represents the height of the track at horizontal position x. Locate the highest and lowest points on the track. (3)

3 A firm produces three products. These products are processed on three different machines. The time required to manufacture one unit of each of the three products and the daily capacity of the three machines are given in the table below. (3)

Machine	Time per unit (minutes)			Machine capacity (minutes/day)
	Product 1	Product 2	Product 3	
M1	2	3	2	440
M2	4	-	3	470
M3	2	5	-	430
Profit per unit	Rs.4	Rs.3	Rs.6	-

It is assumed that all the amounts produced are consumed in the market. Formulate the mathematical model for the problem to maximize the profit.

4 Find the dual of the problem, (3)

$$Max\ Z = 3X_1 + 4X_2$$

Subject to the constraints,

$$2X_1 + 5X_2 \leq 120$$

$$4X_1 + 2X_2 \leq 80$$

$$X_1, X_2 \geq 0$$

5		Explain the key concepts and principles behind the Mini-Max and Maxi-Min theorems in game theory.	(3)
6		Explain minimum spanning tree of a connected graph?	(3)
7		State the importance of the gradient function in minimization problems.	(3)
8		Describe the fundamental principles of the Steepest Descent method for optimization.	(3)
9		List the key differences between crisp sets and fuzzy sets.	(3)
10		Explain the concept of fuzzy logic.	(3)

PART B

Answer any one full question from each module, each carries 14 marks.

Module I

11 a) An engineer is trying to maximize the efficiency of an engine, where efficiency is defined by the objective function $= 10X_1 + 10X_2 - X_1^2 - X_2^2$. However, the engine's operation is subject to certain constraints, (7)

$$X_1 + X_2 \leq 14$$

$$X_1 + X_2 \leq 6$$

$$X_1, X_2 \geq 0$$

Apply KKT conditions to determine the optimal operating conditions that maximize efficiency while respecting the given constraints.

b) Using the method of Lagrange multipliers, solve the following non-linear programming problem, (7)

Optimize $Z = 6X_1^2 + 5X_2^2$

Subject to $X_1 + 5X_2 = 7$

$$X_1, X_2 \geq 0$$

OR

12 a) Find the extreme points of the function, $f(X, Y) = X^2Y - 2XY^2 + 3XY + 4$ and determine whether the points are minimum, maximum or saddle point. (7)

b) A company aims to maximize its profit, which is modelled by the function, (7)

$$f(X) = 2X_1 + X_2 + 10$$

Where X_1 and X_2 represent production levels for two products. However, production is constrained by the resource limitation

$$g(X) = X_1 + 2X_2^2 = 3$$

Using the Lagrange multiplier method, determine the optimal production levels (X_1, X_2) that maximize the company's profit while satisfying the resource constraint.

Module II

13 A company is considering two different production plans, each with its own set of constraints and objective functions represented by the following two LPPs, (14)

(i) $Max\ Z = 5X_1 + 7X_2$

Subject to the constraints,

$$X_1 + X_2 \leq 4$$
$$3X_1 + 8X_2 \leq 24$$
$$10X_1 + 7X_2 \leq 35$$
$$X_1, X_2 \geq 0$$

(ii) $Max\ Z = 7X_1 + 6X_2$

Subject to the constraints,

$$X_1 + X_2 \leq 4$$
$$2X_1 + X_2 \leq 6$$
$$X_1, X_2 \geq 0$$

Analyse the feasible region and explain how changing one of the constraints would impact the optimal solution and the objective function's value. Compare the optimal solutions for each plan. Which plan yields the best results?

OR

14 Given the following Linear Programming Problem, (14)

Maximize $Z = 15X_1 + 6X_2 + 9X_3 + 2X_4$

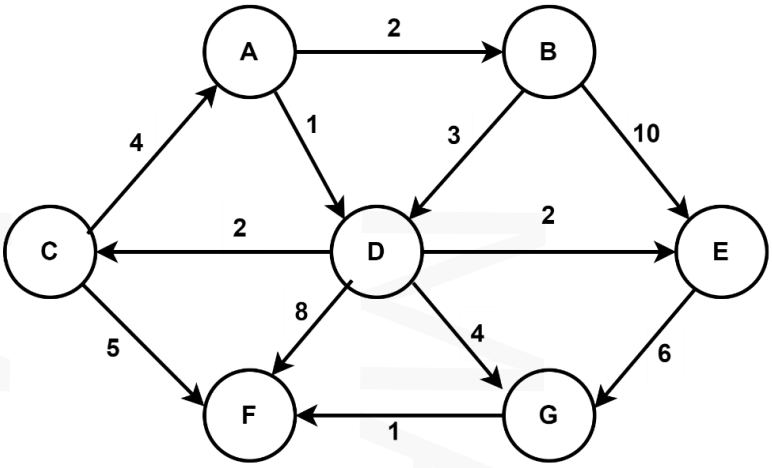
Subject to $2X_1 + X_2 + 5X_3 + 6X_4 \leq 20$

$$3X_1 + X_2 + 3X_3 + 25X_4 \leq 24$$
$$7X_{1+}X_4 \leq 70$$
$$X_1, X_2, X_3, X_4 \geq 0$$

Apply the simplex method to determine the optimal solution.

Module III

15 Use Dijkstra's algorithm to determine the shortest path from node A to all other nodes. Illustrate each step of the algorithm, including the initial distance estimates, the selection of nodes, and the updates to the distance estimates. (14)



OR

16 Given a two-person, zero-sum game with the following payoff matrix (14)

Player B

Player A	2	-1	5	-2	6
	-2	4	-3	1	0

Graphically determine the optimal mixed strategies for both players.

Module IV

17 Determine the minimum point of the function $f(x) = x^2 + \frac{54}{x}$ with the Fibonacci search method. Choose $a = 0, b = 5\ and\ n = 3$. (14)

OR

18 Minimize $f(x, y) = (x^2 + y - 11)^2 + (x + y^2 - 7)^2$ using Hook Jeeves pattern search Method. Choose the initial value as (0,0) and the step size as Δ= (0.5, 0.5). (14)

Module V

19 a) Describe the purpose and function of each of the primary genetic algorithm operators, "Selection, Crossover, and Mutation". (9)

b) List the advantages and disadvantages of using Genetic Algorithms in optimization problems. (5)

OR

20 a) Describe the characteristics of a fuzzy membership function and give an example of how it represents real-world uncertainty. (9)

b) Define the terms "core" and "α-cut" (5)

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