ENTRI

APJ Abdul Kalam Technological University Third Semester B.Tech Degree Examination, December 2020

MAT201: Partial Differential Equations and Complex Analysis.

PART A

(Each question carries 3 marks)

1. Derive a partial differential equation from the relation $z = (x + y)f(x^2 - y^2)$. (p. 1)
2. Solve using direct integration: $\frac{\partial^2 u}{\partial x \partial t} = e^{-t}\cos x$. (p. 1)
3. Solve $2z = xp + yq$. (p. 1)
4. Write any three assumptions in deriving the one-dimensional heat equation. (p. 1)
5. Show that an analytic function $f(z) = u + iv$ is constant if its real part is constant. (p. 1)
6. Show that the function u = sin x cos h y is harmonic. (p. 1)
7. Find the Maclaurin series of $f(z) = \sin z$. (p. 1)
8. Evaluate $\oint_C \ln z\, dz$, where C is the unit circle $|z| = 1$. (p. 1)
9. Find all singular points and the residue of the function csc z. (p. 1)
10. Determine the location and order of zeros of the function $\sin^4(\frac{z}{2})$. (p. 1)

PART B

(Answer any one full question from each module. Each question carries 14 marks)

Module 1

1. (a) Form the Partial differential equation by eliminating the arbitrary constants from $(x - a)^2 + (y - b)^2 = z^2\cot^2\alpha$. (5 marks) (p. 1)
   (b) Solve $2xz - px^2 - 2qxy + pq = 0$. (9 marks) (p. 1)

2. (a) Solve $\frac{\partial^3 z}{\partial^2 x \partial y} = \cos(2x + 3y)$. (7 marks) (p. 1)

   (b) Solve $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$. (7 marks) (p. 1)

Module 2

1. (a) Derive the solution of the one-dimensional wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ using the variable separable method. (6 marks) (p. 1)

   (b) An insulated rod of length *l* has its ends *A* and *B* maintained at $0°C$ and $100°C$ respectively until steady state conditions prevail. If *B* is suddenly reduced to $0°C$ and maintained at $0°C$, find the temperature at a distance *x* from *A* at time *t*. (8 marks) (pp. 1-2)
2. (a) Derive the one-dimensional heat flow equation. (6 marks) (p. 2)

   (b) A tightly stretched string of length *l* with fixed ends is initially in equilibrium position. If it is set vibrating by giving each point a velocity $v_0 \sin^3(\frac{\pi x}{l})$, find the displacement *y*(*x*, *t*). (8 marks) (p. 2)

Module 3

1. (a) Find an analytic function whose real part is *u* = sin *x* cosh *y*. (7 marks) (p. 2)

   (b) Find the image of the strip $\frac{1}{2} \leq x \leq 1$ under the transformation $w = z^2$. (7 marks) (p. 2)
2. (a) Check whether *w* = log *z* is analytic. (8 marks) (p. 2)

   (b) Show that under the transformation $w = \frac{1}{z}$, the circle $x^2 + y^2 - 6x = 0$ is transformed into a straight line in the w plane. (6 marks) (p. 2)

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Module 4

1. (a) Integrate counter-clockwise around the unit circle $\oint_C \frac{\sin 2z}{z^4} dz$. (7 marks) (p. 2)

   (b) Find the Taylor series of $\frac{1}{1+z}$ about the centre $z_0 = i$. (7 marks) (p. 2)

2. (a) Evaluate $\int_0^{1+i} (x - y + ix^2)dz$ along the parabola $y = x^2$. (7 marks) (p. 2)

   (b) Evaluate $\oint_C \frac{\log z}{(z-4)^2} dz$ counter-clockwise around the circle $|z - 3| = 2$. (7 marks) (p. 2)

Module 5

1. (a) Find the Laurent's series expansion of $\frac{z^2 - 1}{z^2 - 5z + 6}$ about z = 0 in the region 2 < |z| < 3. (5 marks) (p. 2)

   (b) Evaluate $\int_0^{2\pi} \frac{d\theta}{\sqrt{2} - \cos\theta}$. (9 marks) (p. 2)

2. (a) Evaluate $\oint_C \frac{z - 23}{z^2 - 4z - 5} dz$ where C : |z - 2 - i| = 3.2 using Residue theorem. (5 marks) (p. 2)

   (b) Evaluate $\int_0^{\infty} \frac{(x^2 + 2)dx}{(x^2+1)(x^2+4)}$. (9 marks) (p. 2)