ENTRI

APJ Abdul Kalam Technological University Third Semester B.Tech Degree Examination, December 2021

MAT201: Partial Differential Equations and Complex Analysis.

PART A

(Each question carries 3 marks) (p. 1)

1. Find the partial differential equation by eliminating arbitrary functions *f* and *g* from *z* = f(x) + g(y).
2. Solve $\frac{\partial^2 z}{\partial x^2} = xy.$.
3. Write the three possible solutions of one dimensional wave equation.
4. Write any two assumptions used in the derivation of one dimensional heat equation.
5. Test the continuity at *z* = 0 of $f(z) = \begin{cases} \frac{Im(z)}{|z|}, & z \neq 0 \\ 0, & z = 0 \end{cases}$.
6. Check whether $f(z) = \bar{z}$ is an analytic function?
7. Evaluate $\oint_C \frac{e^z}{z-5} dz,$, where C is the circle |z| = 4.
8. Find the Taylor series expansion of e^z about $z = \pi$.
9. Give example of (a) removable singularity (b) pole (c) essential singularity.
10. Find the Laurent series expansions of $\frac{1}{z(z-1)}$ about $z = 0$.

PART B

(Answer any one full question from each module. Each question carries 14 marks) (p. 1)

Module 1

1. (a) Solve $y^2 p - xyq = xz$. (7 marks) (p. 1)
   (b) Solve by the method of separation of variables
   $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0, u(x,0) = 4e^{-3x}$. (7 marks) (p. 1)
2. (a) Find the complete integral of px + qy = pq using Charpit's method. (7 marks) (p. 1)
   (b) Form the partial differential equation corresponding to family of spheres with center on z-axis and radius a. (7 marks) (p. 1)

Module 2

1. (a) Solve the boundary value problem described by
   $u_{tt} - c^2 u_{xx} = 0, 0 \leq x \leq \ell, t \geq 0$ with
   $u(0,t) = u(\ell,t) = 0, t \geq 0, u(x,0) = 10\sin(\frac{\pi x}{\ell}), \frac{\partial u}{\partial t}(x,0) = 0$
   . (7 marks) (p. 2)
   (b) Find the temperature u(x, t) in a homogenous bar heat conducting material of length l whose ends kept at $0^\circ C$ and whose initial temperature is given by $u(x, 0) = lx - x^2. (7 marks) (p. 2)
2. (a) Derive one dimensional wave equation. (7 marks) (p. 2)
   (b) The ends A and B of a rod 10 cm in length are kept at temperatures $0^\circ C$ and $100^\circ C$ until the steady state condition prevails. If B is suddenly reduced to $0^\circ C$ and kept so, find the temperature distribution in the rod at time t. (7 marks) (p. 2)

Module 3

1. (a) Show that an analytic function $f(z) = u + iv$ is constant if its modulus is constant. (7 marks) (p. 2)

(b) Find the image of $1 \leq |z| \leq 2, \frac{\pi}{6} \leq \theta \leq \frac{\pi}{3}$ under the mapping $w = z^2$. (7 marks) (p. 2)

2. (a) Verify whether $u = x^3 - 3xy^2$ is harmonic and find its conjugate harmonic function v. (7 marks) (p. 2)
   (b) Find the image of the region between real axis and a line parallel to real axis at $y = \frac{\pi}{2}$ under the mapping $w = e^z$. (7 marks) (p. 2)

Module 4

1. (a) Evaluate $\int_C |z|^2 dz$ where C is the circle |z| = 2. (7 marks) (p. 2)
   (b) Evaluate $\int_C \frac{z^2+2}{(z-3)^2} dz$ where C is the circle |z| = 4 using Cauchy's integral formula. (7 marks) (p. 2)
2. (a) Evaluate $\oint_C \frac{e^z}{(z-1)(z-4)} dz$, where C is |z| = 2 using Cauchy's integral formula. (7 marks) (p. 2)
   (b) Evaluate $\int \frac{3z^2+7z}{z+1} dz$ over (a) |z| = 1.5 (b)|z+i| = 1$. (7 marks) (p. 2)

Module 5

1. (a) Find the Laurent series expansion for $f(z) = \frac{1}{(z-1)(z-2)}$ valid in (a) 1 < |z| < 2 (b) |z| > 2. (7 marks) (p. 2)
   (b) Evaluate $\int_0^{2\pi} \frac{d\theta}{5 - 4\sin\theta}$. (7 marks) (p. 3)

2. (a) Evaluate $\int_{-\infty}^{\infty} \frac{x^2+2}{(x^2+1)(x^2+4)} dx$. (7 marks) (p. 3)

(b) Using residue theorem evaluate $\oint_C \frac{z+1}{z^4-2z^3} dz$, where C is $|z| = \frac{1}{2}$. (7 marks) (p. 3)