1. APJ **Abdul Kalam Technological University Third Semester B.Tech Degree Regular and Supplementary Examination, December 2022**

MAT201: Partial Differential Equations and Complex Analysis

PART A

(Answer all questions. Each question carries 3 marks)

1. Form a partial differential equation from the relation $z = (x + y) f(x^2 - y^2)$ (p. 1).
2. Solve $\dfrac{\partial^3 z}{\partial x^2 \partial y} = \cos(3x + 4y)$ (p. 1).
3. Write any three assumptions involved in the derivation of one dimensional wave equation (p. 1).
4. Find the steady state temperature distribution in a rod of length 25 cm, if the ends of the rod are kept at $20°C$ and $70°C$ (p. 1).
5. Determine whether w = cos z is analytic (p. 1).
6. Check whether the function xy^2 is the real part of an analytic function (p. 1).
7. Using Cauchy's integral formula, Evaluate $\int_C \dfrac{z^2 + 1}{z^2 - 1} dz$ where C is the circle of unit radius with centre at z = 1 (p. 1).
8. Find the Taylor's series of $\dfrac{1}{z}$ about the point z = 1 (p. 1).
9. Find the Laurent series of $z^2 e^{1/z}$ about z = 0 and determine the region of convergence (p. 1).
10. Find the zeros and their order of the function $\sin^2(z)$ (p. 1).

PART B

(Answer any one full question from each module. Each question carries 14 marks)

Module 1

1. (a) Find the differential equation of all planes which are at a constant distance 'c' from the origin (7 marks) (p. 1).

   (b) Solve $y^2p - xyq = x(z - 2y)$ (7 marks) (p. 1).

2. (a) Solve pq + 2x(y+1)p + y(y+2)q - 2(y+1)z = 0 (7 marks) (p. 2).

   (b) Solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ where $u(x,0) = 3e^{5x}$ by method of separation of variables (7 marks) (p. 2).

Module 2

1. (a) A tightly stretched string of length one cm is fastened at both ends. Find the displacement of a string if it is released from rest from the position $\sin \pi x + 5\sin 3\pi x$ (7 marks) (p. 2).

   (b) A rod of 30 cm long has its ends A and B kept at $30°C$ and $90°C$ respectively until steady state temperature prevails. The temperature at each end is then suddenly reduced to zero temperature and kept so. Find the resulting temperature function u(x, t), taking x = 0 at A (7 marks) (p. 2).

2. (a) A tightly stretched homogeneous string of unit length with its fixed ends at x = 0 and x = 1, executes transverse vibrations. The initial velocity is zero and the initial deflection is given by $u(x,0) = \begin{cases} 1, & 0 \le x < \frac{1}{2} \\ -1, & \frac{1}{2} \le x \le 1 \end{cases}$. Find the deflection u(x,t) at any time t (7 marks) (p. 2).

   (b) Derive one dimensional heat equation (7 marks) (p. 2).

Module 3

1. (a) Find the image of the semi circle $y = \sqrt{4 - x^2}$ under the transformation $w = z^2$ (7 marks) (p. 2).

(b) Show that $u = x^2 - y^2 - y$ is harmonic. Also find the corresponding harmonic conjugate function (7 marks) (p. 2).

2. (a) Find the image of the circle |z - 1| = 1 under the mapping $w = \frac{1}{z}$ (7 marks) (p. 2).
   (b) If f(z) = u(x,y) + iv(x,y) is analytic and uv = 2023, then show that f(z) is a constant (7 marks) (p. 2).

Module 4

1. (a) Using Cauchy's integral formula, Evaluate the integral $\int_C \frac{2z+3}{z^2} dz$ is a circle $|z - i| = 2$ counter clockwise (7 marks) (p. 2).
   (b) Evaluate $\int_C (z^2 + 3z) dz$ along the circle |z| = 2 from (2,0) to (0,2) in counter-clockwise direction (7 marks) (p. 2).
2. (a) Using Cauchy's integral formula, Evaluate $\oint_C \frac{e^{2z}}{(z+1)^4} dz$ where C is the circle |z| = 2 (7 marks) (p. 3).
   (b) Expand $f(z) = \frac{z+1}{z-1}$ as a Taylor series about z = -1 (7 marks) (p. 3).

Module 5

2. (a) Find the Laurent series expansion of $f(z) = \frac{1}{1-z^2}$ about z = 1 in the regions (i) 0 < |z - 1| < 2 (ii) |z - 1| > 2 (7 marks) (p. 3).
   (b) Evaluate $\int_0^{2\pi} \frac{1}{2 + \cos\theta} d\theta$ (7 marks) (p. 3).

ENTRI

3. (a) Using Cauchy's Residue theorem, Evaluate $\int_C \frac{30z^2 - 23z + 5}{(2z-1)^2(3z-1)} dz$ where C is the circle |z| = 1 counter-clockwise (7 marks) (p. 3).

   (b) Using contour integration, evaluate $\int_{-\infty}^{\infty} \frac{1}{(1+x^2)^3} dx$ (7 marks) (p. 3).