APJ Abdul Kalam Technological University B.Tech Degree Examination, June 2024

MAT201: Partial Differential Equations and Complex Analysis.

PART A

(Answer all questions. Each question carries 3 marks)

1. Form the partial differential equation by eliminating arbitrary function from the relation $z = x - y + f(x^2 + y^2)$ (p. 1).
2. Solve by direct integration: $\frac{\partial^2 z}{\partial x \partial y} = \sin(3x + 4y)$ (p. 1).
3. Write all possible solutions of one-dimensional wave equation (p. 1).
4. Write any three assumptions used in the derivation of one-dimensional heat equation (p. 1).
5. Check whether $f(z) = z - \bar{z}$ is analytic or not (p. 1).
6. Test the continuity of f(z) at the point z = 0, where $f(z) = \begin{cases} \frac{Imz}{|z|}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ (p. 1).
7. Find the value of the integral $\int_C z\,dz$ where C is the part of the unit circle from -i to i in the right half plane (p. 1).
8. Evaluate $\oint_C \frac{\cos z}{z^2 + 9} dz$, where C is the circle |z| = 1 (p. 1).
9. Find the Laurent series expansion of $\frac{1}{1 - z^2}$ about z = 0 (p. 1).

10. Find the singularities and nature of singularities of the function $f(z) = z^2 \cos \frac{1}{z}$ (p. 1).

PART B

(Answer any one full question from each module. Each question carries 14 marks)

Module 1

1. (a) Solve the PDE $x(yz - z^2)p + y(z^2 - xz)q = z(x^2 - y^2)$ (7 marks) (p. 1).
   (b) Solve by Method of Separation of variables $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial t} = 0, u(x, 0) = 4e^{-3x}$ (7 marks) (p. 1).
2. (a) Solve $2xz - px^2 - 2qxy + pq$ (7 marks) (p. 1).
   (b) Solve $(y^2 + z^2)p - xyq = xz$ (7 marks) (p. 1).

Module 2

1. (a) A tightly stretched string with fixed end points x = 0 and x = l is initially at rest in its equilibrium position. If it is vibrating by giving each of its points a velocity $\lambda x(l - x)$, find the displacement of the string at any distance x from one end at any time t. (7 marks) (p. 2).
   (b) Derive One Dimensional Wave equation (7 marks) (p. 2).
2. (a) Derive all possible solutions of one-dimensional wave equation (7 marks) (p. 2).
   (b) Find the temperature u(x, t) in a homogeneous rod of length 2m, whose ends are kept at $0°C$ and whose initial temperature is given by $f(x) = 100(2x - x^2), 0 \leq x \leq 2$ (7 marks) (p. 2).

Module 3

1. (a) Show that $f(z) = e^z$ is analytic for all z in the complex plane and find its derivative (7 marks) (p. 2).
   (b) Find image of the circle $|z - \frac{1}{2}| \leq \frac{1}{2}$ under the transformation $w = \frac{1}{z}$ (7 marks) (p. 2).
2. (a) Show that the function u = cos x cosh y is harmonic and find its harmonic conjugate (7 marks) (p. 2).
   (b) Show that an analytic function f(z) = u + iv is constant if |f(z)| is constant (7 marks) (p. 2).

Module 4

1. (a) Evaluate $\int_C \frac{z+1}{z^2-2z}\,dz$ where |z| = 1 (7 marks) (p. 2).
   (b) Find the Taylor series expansion of $f(z) = \sin 2z$ about $z = \frac{\pi}{2}$ (7 marks) (p. 2).
2. (a) Evaluate $\int_C |z|\,dz$ where C is the part of the circle |z| = 3 from 3 to -3 in the upper half plane (7 marks) (p. 2).
   (b) Evaluate $\int_C \frac{e^{3z}}{(z+1)^4}\,dz$ where C is the circle |z - 1| = 3 (7 marks) (p. 2).

Module 5

1. (a) Find Laurent series expansion $f(z) = \frac{-2z+3}{(z^2-3z+2)}$ valid in: (i) $1 \leq |z| \leq 2$ (ii) |z| $\geq$ 2 (7 marks) (p. 2).
   (b) Evaluate $\oint_C \frac{e^z}{z^2(z+1)}\,dz$ over the circle |z| = 2 (7 marks) (p. 2).

2. (a) Evaluate $\int_C \tan z \, dz$ where C is circle |z| = 2 (7 marks) (p. 2).

   (b) Evaluate $\int_0^{2\pi} \frac{d\theta}{\sqrt{2} - \cos\theta}$ (7 marks) (p. 2).