Reg No.:_______________ Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Third Semester B.Tech Degree Examination December 2020 (2019 Scheme)

Course Code: MAT201

Course Name: Partial Differential equations and Complex analysis

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions. Each question carries 3 marks Marks

1 Derive a partial differential equation from the relation z = (x+ y) f (x^2 - y^2) (3)

2 Solve using direct integration $\frac{\partial^2 u}{\partial x \partial t} = e^{-t} cosx$ (3)

3 Solve 2z=xp +yq. (3)

4 Write any three assumptions in deriving one dimensional heat equation. (3)

5 Show that an analytic function f(z) = u+ iv is constant if its real part is constant. (3)

6 Show that the function u = sin x cos hy is harmonic. (3)

7 Find the Maclaurin series of f(z) = sinz (3)

8 Evaluate $\oint_C \ln z\, dz$, where C is the unit circle $|z|$ = 1. (3)

9 Find all singular points and residue of the function cosec z (3)

10 Determine the location and order of zeros of the function $sin^4 (\frac{z}{2})$ (3)

PART B

Answer any one full question from each module. Each question carries 14 marks

Module 1

11 (a) Form the Partial differential equation by eliminating the arbitrary constants from $(x - a)^2 + (y - b)^2 = z^2 cot^2$ α (5)

(b) Solve 2xz –p x^2 - 2qxy + pq = 0 (9)

12 (a) Solve $\frac{\partial^3 z}{\partial^2 x \partial y}$ = Cos (2x+ 3 y) (7)

(b) Solve x^2 (y – z)p + y^2 (z- x) q = z^2 (x – y) (7)

Module 2

13 (a) Derive the solution of the one dimensional wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ using variable separable method. (6)

(b) An insulated rod of length l has its ends A and B maintained at 0^0 C and

100^0 C respectively until steady state conditions prevail. If B is suddenly reduced to 0^0 C and maintained at 0^0 C, find the temperature at a distance x from A at time t. (8)

14 (a) Derive the one dimensional heat flow equation. (6)

(b) A tightly stretched string of length l with fixed ends is initially in equilibrium position .If it is set vibrating by giving each points a velocity $v_0 sin^3(\frac{\pi x}{l})$. Find the displacement y(x,t). (8)

Module 3

15 (a) Find an analytic function whose real part is u = sinx coshy (7)

(b) Find the image of the strip $\frac{1}{2} \le x \le 1$ under the transformation w = z^2 (7)

16 (a) Check whether w = logz is analytic. (8)

(b) Show that under the transformation w = $\frac{1}{z}$, the circle $x^2 + y^2 - 6x = 0$ is transformed into a straight line in the W plane. (6)

Module 4

17 (a) Integrate counter clockwise around the unit circle $\oint_C \frac{\sin 2z}{z^4}$ dz (7)

(b) Find the Taylor series of $\frac{1}{1+z}$ about the centre z_0 = i (7)

18 (a) Evaluate $\int_0^{1+i}(x - y + ix^2)$ dz along the parabola y= x^2. (7)

(b) Evaluate $\oint_C \frac{\log z}{(z-4)^2}$ dz counter clockwise around the circle $|z - 3|$=2. (7)

Module 5

19 (a) Find the Laurent's series expansion of $\frac{z^2 - 1}{z^2 - 5z + 6}$ about z= 0 in the region 2< $|z| < 3$ (5)

(b) Evaluate $\int_0^{2\pi} \frac{d\theta}{\sqrt{2} - cos\theta}$. (9)

20 (a) Evaluate $\oint_C \frac{z-23}{z^2 - 4z - 5}$ dz where C : $|z - 2 - i|$ = 3.2 using Residue theorem. (5)

(b) Evaluate $\int_0^{\infty} \frac{(x^2 + 2)dx}{(x^2 + 1)(x^2 + 4)}$. (9)

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