Reg No.:_______________ Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Third Semester B.Tech Degree Examination December 2021 (2019 scheme)

Course Code: MAT201

Course Name: Partial Differential equations and Complex analysis

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions. Each question carries 3 marks Marks

1 Find the partial differential equation by eliminating arbitrary functions f and g from z = f(x) + g(y) (3)

2 Solve $\frac{\partial^2 z}{\partial x^2}$ = xy (3)

3 Write the three possible solutions of one dimensional wave equation. (3)

4 Write any two assumptions used in the derivation of one dimensional heat equation. (3)

5 Test the continuity at $z = 0$ of $f(z) = \begin{cases} \frac{Im(Z)}{|z|}, z \neq 0 \\ 0, \quad z = 0 \end{cases}$ (3)

6 Check whether f (z) = $\bar{z}$ is an analytic function? (3)

7 Evaluate $\oint_c \frac{e^z}{z-5}\, dz$, where c is the circle $|z|$= 4 (3)

8 Find the Taylor series expansion of e^z about z = π. (3)

9 Give example of (a) removable singularity (b) pole (c) essential singularity (3)

10 Find the Laurent series expansions of $\frac{1}{z(z-1)}$ about $z = 0$ (3)

PART B

Answer any one full question from each module. Each question carries 14 marks

Module 1

11 (a) Solve $y^2p - xyq = xz$ 7

(b) Solve by the method of separation of variables $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}$ =0, u(x, 0) = $4e^{-3x}$ 7

12 (a) Find the complete integral of $px + qy = pq$ using Charpit's method. 7

(b) Form the partial differential equation corresponding to family of spheres with center on z-axis and radius a. 7

Module 2

13 (a) Solve the boundary value problem described by $u_{tt}-c^2u_{xx}=0$, $0 \le x \le \ell$, $t\ge0$ 7

$u(0, t) = u(\ell,t)=0$, $t\ge0$, $u(x, 0) = 10 \sin\left(\frac{\pi x}{\ell}\right)$, $\frac{\partial u}{\partial t}(x, 0) = 0$

(b) Find the temperature $u(x,t)$ in a homogenous bar heat conducting material of length l whose ends kept at $0°c$ and whose initial temperature is given by $u(x,0) = lx - x^2$. 7

14 (a) Derive one dimensional wave equation. 7

(b) The ends A and B of a rod 10 cm in length are kept at temperatures 0°C and 100°C until the steady state condition prevails. If B is Suddenly reduced to 0°C and kept so . Find the temperature distribution in the rod at time t. 7

Module 3

15 (a) Show that an analytic function $f(z) = u + iv$ is constant if its modulus is constant. 7

(b) Find the image of $1 \le |z| \le 2$, $\frac{\pi}{6} \le \theta \le \frac{\pi}{3}$ under the mapping $w = z^2$ 7

16 (a) Verify whether $u = x^3 - 3xy^2$ is harmonic and find its conjugate harmonic function v. 7

(b) Find the image of the region between real axis and a line parallel to real axis at $y = \frac{\pi}{2}$ under the mapping $W= e^z$ 7

Module 4

17 (a) Evaluate $\int_C |z|^2\, dz$ where C is the circle $|z| = 2$. 7

(b) Evaluate $\int_C \frac{z^2+2}{(z-3)^2} dz$ where C is the circle $|z| = 4$ using Cauchy's integral formula 7

18 (a) Evaluate $\oint_c \frac{e^z}{(z-1)(z-4)}$ dz, where c is $|z| = 2$ using Cauchy's integral formula 7

(b) Evaluate $\int \frac{3z^2+7z}{z+1}$ dz over (a) $|z| = 1.5$ (b) $|z + i| = 1$ 7

Module 5

19 (a) Find the Laurent series expansion for $f(z) = \frac{1}{(z-1)(z-2)}$ valid in 7

(a) $1 < |z| < 2$ (b)) $|z| > 2$

7

(b) Evaluate $\int_0^{2\pi} \frac{d\theta}{5-4\sin\theta}$

20 (a) Evaluate $\int_{-\infty}^{\infty} \frac{x^2+2}{(x^2+1)(x^2+4)} dx$ 7

(b) Using residue theorem evaluate $\oint_c \frac{z+1}{z^4-2z^3}$ dz ,where c is $|z| = \frac{1}{2}$ 7

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