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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Fourth Semester B.Tech Degree Examination July 2021 (2019 Scheme)

Course Code: MAT204

Course Name: PROBABILITY, RANDOM PROCESSES AND NUMERICAL METHODS

Max. Marks: 100 Duration: 3 Hours

(Statistical Tables are allowed)

PART A

(Answer all questions; each question carries 3 marks) Marks

1	In a binomial distribution, if the mean is 6, and the variance is 4, find P[X=1].	3
2	Given that f(x) = $\frac{K}{2^x}$ is a probability mass function of a random variable that can take on the values x = 0,1,2,3 and 4, find (i) K and (ii) $P(X \leq 2)$.	3
3	Find the mean and variance for the PDF, f(x) = $\begin{cases} Kx^2 & ,0 < X < 1 \\ 0 & ,elsewhere \end{cases}$	3
4	If random variable X has a uniform distribution in (-3,3), find P ($\lvert X - 2 \rvert < 2$).	3
5	Define stationary random process. Define two types of stationary random process.	3
6	Write down the properties of the power spectral density.	3
7	Write down the Newton's forward and backward difference interpolation formula	3
8	Evaluate $\int_0^1 \frac{1}{1+x^2}dx$ with 4 subintervals by Simpson's rule.	3
9	Write the normal equations for fitting a curve of the form y = a + bx + cx^2 to a given set of pairs of data points.	3
10	Using Euler's method, find y at x = 0.25, given y' = 2xy, y (0) =1, h= 0.25.	3

PART B

(Answer one full question from each module, each question carries 14 marks)

Module -1

11	a)	Six dice are thrown 729 times. How many times do you expect at least three dice to show 1 or 2?	6
	b)	Derive the formula for mean and variance of Poisson distribution	8
12	a)	A random variable X takes the values -3, -2, -1,0,1,2,3 such that P(X=0) = P(X>0) = P(X<0) and P(X=-3) = P(X=-2) = P(X=-1) = P(X=1) = P(X=2) = P(X=3). Obtain the probability mass function and distribution function of X.	7

b) The joint probability distribution of X and Y is given by, $f(x,y) = \frac{1}{27}(2x + y)$; 7

$x = 0,1,2$ and $y = 0,1,2$.

(i) Find the marginal distributions of X and Y.

(ii) Are X and Y independent random variables.

Module -2

13 a) Suppose the diameter at breast height (in.) of trees of a certain type is normally distributed with mean 8.8 and standard deviation 2.8.(i) What is the probability that the diameter of a randomly selected tree will be at least 10 in.? (ii) What is the probability that the diameter of a randomly selected tree will exceed 20 in.? (iii) What is the probability that the diameter of a randomly selected tree will be between 5 in. and 10 in.? 7

b) The amount of time that a surveillance camera will run without having to be reset is a random variable having exponential distribution with mean 50 days. Find the probabilities that such a camera will (a) have to be reset in less than 20 days. (b) not have to be reset in at least 60 days. 7

14 a) The joint density function of 2 continuous random variable X and Y is 7

$$f(x,y) = \begin{cases} cxy \ ; & 0 < x < 4, \ 1 < y < 5 \\ 0 \ ; & otherwise \end{cases}$$

(i) Find the value of the constant c.

(ii) Find $P(X \geq 3, Y \leq 2)$

(iii) Find the marginal density of X.

b) The life time of a certain brand of tube light may be considered as a random variable with mean 1200 hours and standard deviation 250 hours. Using Central limit theorem, find the probability that the average life time of 60 lights exceeds 1250. 7

Module -3

15 a) Let $X(t) = A \cos \lambda t + B \sin \lambda t$, where A and B are independent normally distributed random variables $N(0,\sigma^2)$. Show that X(t) is WSS. 7

b) If $X(t) = A \cos(\omega t + \theta)$ Where A and ω are constants and θ is uniformly distributed over $[0,2\pi]$, find the auto correlation function and Power Spectral Density of the process. 7

16 a) Assume that X(t) is a random process defined as follows: $X(t) = A\cos(2\pi t + \emptyset)$ where A is a zero-mean normal random variable with variance $\sigma_A^2 = 2$ and $\emptyset$ is uniformly distributed random variable over the interval $-\pi \le \varphi \le \pi$. A and φ are statistically independent. Let the random variable Y be defined as $Y = \int_0^1 X(t)dt$. Determine (i) The mean of Y (ii) The variance of Y. 7

b) If the customers arrive at a counter in accordance with Poisson distribution with rate of 2 per minute. Find the probability that the interval between two consecutive arrivals is (i) more than 1 minute (ii) between 1 minute and 2 minutes. 7

Module -4

17 a) Use Newton- Raphson method to find a non- zero solution of f(x)=2x - Cos x = 0 7

b) Using Lagrange's interpolating polynomial estimate y (5) for the following data: 7

x	1	3	4	6
y	-3	0	30	132

18 a) Find the polynomial interpolating the following data, using Newton's backward interpolating formula 7

x	3	4	5	6	7
y	7	11	16	22	29

b) Using Newton's divided difference formula, evaluate y(8) and y(15) from the following data 7

x	4	5	7	10	11	13
y	48	100	294	900	1210	2028

Module -5

19 a) Solve the following system of equations using Gauss- Seidel iteration method starting with the initial approximation $(0,0,0)^T$ 7

$$8x_1 + x_2 + x_3 = 8$$

$$2x_1 + 4x_2 + x_3 = 4$$

$$x_1 + 3x_2 + 5x_3 = 5$$

b) Fit a straight-line y = ax + b for the following data: 7

X	1	3	4	6	8	9	11	14
Y	1	2	4	4	5	7	8	9

20 a) Solve the following system of equations using Gauss- Jacobi iteration method starting with the initial approximation $(0,0,0)^T$ 7

$$20x_1 + x_2 - 2x_3 = 17$$
$$3x_1 + 20x_2 - x_3 = -18$$
$$2x_1 - 3x_2 + 20x_3 = 25$$

b) Use Runge - Kutta method of fourth order to find y (0.1) from $\frac{dy}{dx} = \sqrt{x+y}$, y (0) =1 taking h = 0.1. 7

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