APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Fourth Semester B.Tech Degree Examination July 2021 (2019 Scheme)

Course Code: MAT204

Course Name: PROBABILITY, RANDOM PROCESSES AND NUMERICAL METHODS

Max. Marks: 100 Duration: 3 Hours

(Statistical Tables are allowed) (p. 1)

PART A

Answer all questions; each question carries 3 marks

1. In a binomial distribution, if the mean is 6, and the variance is 4, find $P[X = 1]$.
2. Given that $f(x) = \frac{K}{2^x}$ is a probability mass function of a random variable that can take on the values x = 0,1,2,3 and 4, find (i) K and (ii) $P(X \leq 2)$.
3. Find the mean and variance for the PDF,

$$f(x) = \begin{cases} Kx^2 & , 0 < X < 1 \\ 0 & , \text{elsewhere} \end{cases}$$

4. If random variable X has a uniform distribution in (-3,3), find P(|X - 2| < 2).
5. Define stationary random process. Define two types of stationary random process.
6. Write down the properties of the power spectral density.
7. Write down the Newton's forward and backward difference interpolation formula.

8. Evaluate $\int_0^1 \frac{1}{1+x^2}\,dx$ with 4 subintervals by Simpson's rule.
9. Write the normal equations for fitting a curve of the form $y = a + bx + cx^2$ to a given set of pairs of data points.
10. Using Euler's method, find y at x = 0.25, given $y' = 2xy, y(0) = 1, h = 0.25$. (p. 1)

PART B

Answer one full question from each module, each question carries 14 marks

Module - 1

11. (a) Six dice are thrown 729 times. How many times do you expect at least three dice to show 1 or 2? (6 marks)

(b) Derive the formula for mean and variance of Poisson distribution. (8 marks)

12. (a) A random variable X takes the values -3, -2, -1, 0, 1, 2, 3 such that P(X=0) = P(X>0) = P(X<0) and P(X=-3) = P(X=-2) = P(X=-1) = P(X=1) = P(X=2) = P(X=3). Obtain the probability mass function and distribution function of X. (7 marks) (p. 1)

Page 2

1. (b) The joint probability distribution of X and Y is given by, $f(x,y) = \frac{1}{27}(2x+y)$; x = 0,1,2 and y = 0,1,2.
   (i) Find the marginal distributions of X and Y.
   (ii) Are X and Y independent random variables. (7 marks) (p. 2)

Module - 2

13. (a) Suppose the diameter at breast height (in.) of trees of a certain type is normally distributed with mean 8.8 and standard deviation 2.8. (i) What is the probability that the diameter of a randomly selected tree will be at least 10 in.? (ii) What is the probability that the diameter of a randomly selected tree will exceed 20 in.? (iii) What is the probability that the diameter of a randomly selected tree will be between 5 in. and 10 in.? (7 marks)

(b) The amount of time that a surveillance camera will run without having to be reset is a random variable having exponential distribution with mean 50 days. Find the probabilities that such a camera will (a) have to be reset in less than 20 days. (b) not have to be reset in at least 60 days. (7 marks) (p. 2)

14. (a) The joint density function of 2 continuous random variable X and Y is $f(x,y) = \begin{cases} cxy & ; 0 < x < 4, 1 < y < 5 \\ 0 & ; \text{otherwise} \end{cases}$

(i) Find the value of the constant c.

(ii) Find $P(X \geq 3, Y \leq 2)$

(iii) Find the marginal density of X. (7 marks)

(b) The life time of a certain brand of tube light may be considered as a random variable with mean 1200 hours and standard deviation 250 hours. Using Central limit theorem, find the probability that the average life time of 60 lights exceeds 1250. (7 marks) (p. 2)

Module - 3

15. (a) Let $X(t) = A\cos\lambda t + B\sin\lambda t$**, where A and B are independent normally distributed random variables** $N(0, \sigma^2)$**. Show that X(t) is WSS. (7 marks)**

(b) If $X(t) = A\cos(\omega t + \theta)$ Where A and ω are constants and θ is uniformly distributed over $[0, 2\pi]$, find the auto correlation function and Power Spectral Density of the process. (7 marks) (p. 2)

Page 3

1. (a) Assume that X(t) is a random process defined as follows: $X(t) = A\cos(2\pi t + \phi)$ where A is a zero-mean normal random variable with variance $\sigma_A^2 = 2$ and ϕ is uniformly distributed random variable over the interval $-\pi \leq \phi \leq \pi$. A and ϕ are statistically independent. Let the random variable Y be defined as $Y = \int_0^1 X(t)dt$.
   Determine (i) The mean of Y (ii) The variance of Y. (7 marks)
   (b) If the customers arrive at a counter in accordance with Poisson distribution with rate of 2 per minute. Find the probability that the interval between two consecutive arrivals is (i) more than 1 minute (ii) between 1 minute and 2 minutes. (7 marks) (p. 3)

Module - 4

17. (a) Use Newton-Raphson method to find a non-zero solution of f(x) = 2x - cos x = 0. (7 marks)

(b) Using Lagrange's interpolating polynomial estimate y(5) for the following data: (7 marks)

x	1	3	4	6

y	-3	0	30	132

1. (a) Find the polynomial interpolating the following data, using Newton's backward interpolating formula (7 marks)

x	3	4	5	6	7
y	7	11	16	22	29
(b) Using Newton's divided difference formula, evaluate y(8) and y(15) from the following data (7 marks)					

2.

x	4	5	7	10	11	13

y	48	100	294	900	1210	2028

Module - 5

19. (a) Solve the following system of equations using Gauss-Seidel iteration method starting with the initial approximation $(0,0,0)^T$: (7 marks)

$$
\begin{aligned}
8x_1 + x_2 + x_3 &= 8 \\
2x_1 + 4x_2 + x_3 &= 4 \\
x_1 + 3x_2 + 5x_3 &= 5
\end{aligned}
$$

(b) Fit a straight-line y = ax + b for the following data: (7 marks)

X	1	3	4	6	8	9	11	14
Y	1	2	4	4	5	7	8	9

Page 4

1. **(a) Solve the following system of equations using Gauss-Jacobi iteration method starting with the initial approximation $(0,0,0)^T$: (7 marks)**

$$
\begin{aligned}
20x_1 + x_2 - 2x_3 &= 17 \\
3x_1 + 20x_2 - x_3 &= -18 \\
2x_1 - 3x_2 + 20x_3 &= 25
\end{aligned}
$$

ENTRI

2. (b) Use Runge-Kutta method of fourth order to find $\frac{dy}{dx} = \sqrt{x+y}$, y(0) = 1 taking h = 0.1. (7 marks) (p. 4)